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Weyl Invariant Formulation of Flux-Tube Solution in the Dual Ginzburg-Landau Theory

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Abstract

The flux-tube solution in the dual Ginzburg-Landau (DGL) theory in the Bogomol'nyi limit is studied by using the manifestly Weyl invariant form of the DGL Lagrangian. The dual gauge symmetry is extended to $[U(1)]^3_m$, and accordingly, there appear three different types of the flux-tube. The string tension for each flux-tube is calculated analytically and is found to be the same owing to the Weyl symmetry. It is suggested that the flux-tube can be treated in quite a similar way with the Abrikosov-Nielsen-Olesen vortex in the $U(1)$ Abelian Higgs theory except for various types of flux-tube.

Key Word: Dual Ginzburg-Landau theory, Weyl symmetry, flux-tube, Bogomol'nyi limit

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I. INTRODUCTION

Recent studies in lattice QCD in the maximally Abelian gauge suggest remarkable properties of the QCD vacuum, such as Abelian dominance [1] and monopole condensation [2], which provide the dual superconductor picture of the QCD vacuum as is described by the dual Ginzburg-Landau (DGL) theory [3,4]. The DGL theory is obtained by using the Abelian projection [5]. In this scheme, QCD is reduced into the $[U(1)]^2$ gauge theory including color-magnetic monopoles. Based on the dual superconductor picture of the QCD vacuum, we get an intuitive picture of hadrons as the vortex excitation of the color-electric flux [6,7], which we call the color-electric flux-tube, or simply the flux-tube. In this vacuum, the color-electric flux is squeezed into an almost one dimensional object like a string due to the dual Meissner effect caused by monopole condensation. This situation seems to be the same with the appearance of the Abrikosov vortex in the ordinary superconductor system, which is caused by the Cooper pair condensation.

We know that Abrikosov-Nielsen-Olesen (ANO) vortex in the ordinary superconductor can be described by using the Abelian Higgs theory [8], where the keyword is the breaking of $U(1)_e$ gauge symmetry through the Higgs mechanism. Moreover, there exists an analytic solution of the ANO vortex in the border of the type-I and the type-II vacuum, so called the Bogomol'nyi limit [9,10]. The analytical solution exhibits interesting features of the superconductivity and is useful to understand the properties of the vortex dynamics. Hence, it is considered quite interesting to investigate the flux-tube solution in the dual superconductor QCD vacuum corresponding to the ANO vortex in the Bogomol'nyi limit.

However, the symmetry in the QCD vacuum is not so simple compared with the ordinary superconductor system, since now we have to take into account the $[U(1)]_m^2$ dual gauge symmetry corresponding to the $U(1)_e$ gauge symmetry in the ordinary superconductor. Note that the symmetry $[U(1)]^{N-1}$ is originated from the maximal torus subgroup of $SU(N)$. Furthermore, we also have the *Weyl* symmetry, which is the permutation invariance of the labels among the Abelian color charges. Therefore, the flux-tube in the QCD vacuum is expected to have some characteristic aspects beyond the analogue of the ANO vortex in the ordinary superconductor system.

In this paper, we investigate the flux-tube solution in the DGL theory in the Bogomol'nyi limit. This study seems to be similar as is given in Ref. [11]. In fact, our result will be shown identical. However, we would like to present an usuful method to find the Bogomol'nyi limit,

and this can be achieved by taking into account the Weyl symmetry in the DGL theory. This idea can be extended straightforwardly to the $[U(1)]^{N-1}$ dual Abelian Higgs theory which would be reduced from the $SU(N)$ gluodynamics [12]. We first write the DGL Lagrangian in a manifestly Weyl invariant form. At the same time, we pay attention to the singular structure in the DGL theory, since it plays a significant role to obtain the string-like flux-tube solution. Note that the boundary condition of the dual gauge field depends crucially on this singular structure. Second, we consider the Bogomol'nyi limit, the border between the type-I and the type-II vacuum. The string tension in this limit is computed analytically. Finally, we discuss the properties of the flux-tube solution in the DGL theory.

II. MANIFESTLY WEYL INVARIANT FORM OF THE DGL LAGRANGIAN

The DGL Lagrangian [3,4] is given by [1]

$$
\begin{aligned}
\mathcal{L}_{\rm DGL} = & -\frac{1}{4}\left(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu - \frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha \vec{j}^\beta\right)^2 \\
& + \sum_{i=1}^{3}\left[\left|\left(\partial_\mu + ig\vec{\epsilon}_i\cdot\vec{B}_\mu\right)\chi_i\right|^2 - \lambda\left(|\chi_i|^2 - v^2\right)^2\right],
\end{aligned}
\tag{2.1}
$$

where $\vec{B}_\mu$ and χ_i denote the dual gauge field with two components (B^3_μ, B^8_μ) and the complex scalar monopole field, respectively. The quark field is included in the current $\vec{j}_\mu = e\bar{q}\gamma_\mu \vec{H} q$, $\vec{H} = (T_3, T_8)$. Here, $\vec{\epsilon}_i$ is the root vector of $SU(3)$ algebra, $\vec{\epsilon}_1 = \left(-1/2, \sqrt{3}/2\right), \vec{\epsilon}_2 = \left(-1/2, -\sqrt{3}/2\right), \vec{\epsilon}_3 = (1,0)$, and n^μ denotes an arbitrary constant 4-vector[2], which corresponds to the direction of the Dirac string. The gauge coupling e and the dual gauge coupling g hold the relation $eg = 4\pi$. This relation guarantees the unobservability of the Dirac string when the dual gauge symmetry is not broken. Note that the DGL Lagrangian (2.1) is invariant under the $[U(1)]^2_m$ dual gauge transformation,

$$
\chi_i \to \chi_i e^{if_i}, \quad \chi_i^* \to \chi_i^* e^{-if_i},
$$

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[1] Throughout this paper, we use the following notations: Latin indices i,j express the labels 1,2,3, which is not to be summed over unless explicitly stated. Boldface letter denotes three-vector.

[2] If the dual gauge symmetry is broken through monopole condensation, n^μ can not be an arbitrary vector any more. Instead, this vector describes the dynamics of the string and gives the contribution to the energy of the system.

$$\vec{B}_\mu = (B_\mu^3,\ B_\mu^8) \to \left(B_\mu^3 - \frac{1}{g}\partial_\mu f_3,\ B_\mu^8 - \frac{1}{\sqrt{3}g}(\partial_\mu f_1 - \partial_\mu f_2) \right), \qquad (i = 1, 2, 3), \tag{2.2}$$

where the phase f_i has the constraint $\sum_{i=1}^3 f_i = 0$ [3,4].

The non-local term, in the kinetic term of the dual gauge field, is concretely written as

$$\frac{1}{n \cdot \partial}\varepsilon_{\mu\nu\alpha\beta} n^\alpha \vec{j}^\beta = \int d^4x' \langle x | \frac{1}{n \cdot \partial} | x' \rangle \varepsilon_{\mu\nu\alpha\beta} n^\alpha \vec{j}^\beta(x'), \tag{2.3}$$

where

$$\langle x | \frac{1}{n \cdot \partial} | x' \rangle = [p\theta((x - x') \cdot n) - (1 - p)\theta((x' - x) \cdot n)]\, \delta^{(3)}(\vec{x}_\perp - \vec{x}'_\perp). \tag{2.4}$$

Here p is an arbitrary real number and $\delta^{(3)}(x)$ is the δ-function defined on a three dimensional hyper-surface which has the normal vector n_μ, so that $\vec{x}_\perp$ and $\vec{x}'_\perp$ are 3-vectors (not necessarily spatial) which are perpendicular to the n_μ. It is noted that in order to define the color-electric charge of the quark in terms of the *dual* gauge field, we need such a non-local term, which is a result of the choice of *one potential approach* [13].

Now, we define an extended dual gauge field to take into account the Weyl invariance in the DGL theory as

$$B_{i\,\mu} \equiv \sqrt{\frac{2}{3}} \vec{\epsilon}_i \cdot \vec{B}_\mu, \qquad (i = 1, 2, 3) \tag{2.5}$$

Here, the constraint $\sum_{i=1}^3 B_{i\,\mu} = 0$ appears, which has the same structure with the constraint $\sum_{i=1}^3 f_i = 0$. Furthermore, we divide the dual gauge field into two parts, the regular part and the singular part [14,15],

$$\vec{B}_\mu \equiv \vec{B}_\mu^{\rm reg} + \vec{B}_\mu^{\rm sing}. \tag{2.6}$$

The factor $\sqrt{2/3}$ in (2.5) is a simple normalization to get the factor $1/4$ in front of the kinetic term of the dual gauge field (See (2.8)). The singular dual gauge field $\vec{B}_\mu^{\rm sing}$ is determined so as to cancel the Dirac string in the non-local term as

$$\partial_\mu \vec{B}_\nu^{\rm sing} - \partial_\nu \vec{B}_\mu^{\rm sing} - \frac{1}{n \cdot \partial}\varepsilon_{\mu\nu\alpha\beta} n^\alpha \vec{j}^\beta \equiv \vec{C}_{\mu\nu}. \tag{2.7}$$

In the static q-$\bar{q}$ system, $\vec{C}_{\mu\nu}$ is nothing but the color-electric field originated from the color-electric charge like the electric field induced by an electric charge, where an explicit form of $\vec{B}_\mu^{\rm sing}$ is given in Sec. III. It is noted that the cross term of the regular dual field tensor ${}^*\vec{F}_{\mu\nu}^{\rm reg} \equiv \partial_\mu \vec{B}_\nu^{\rm reg} - \partial_\nu \vec{B}_\mu^{\rm reg}$ and $\vec{C}_{\mu\nu}$ can be integrated out, and the square of

$\vec{C}_{\mu\nu}$ and its integration gives the Coulomb energy including the self-energy of the color-electric charge. However, we drop it hearafter in order to concentrate on the flux-tube itself. Correspondingly, we pay attention to the string tension for an ideal flux-tube system which has terminals at infinity[3].

Then, we obtain

$$\mathcal{L}_{\rm DGL} = \sum_{i=1}^{3} \left[-\frac{1}{4} {}^*F^{\rm reg\ 2}_{i\ \mu\nu} + \left| \left(\partial_\mu + ig' \left(B^{\rm reg}_{i\ \mu} + B^{\rm sing}_{i\ \mu} \right) \right) \chi_i \right|^2 - \lambda \left(|\chi_i|^2 - v^2 \right)^2 \right], \tag{2.8}$$

$${}^*F^{\rm reg}_{i\ \mu\nu} \equiv \partial_\mu B^{\rm reg}_{i\ \nu} - \partial_\nu B^{\rm reg}_{i\ \mu}, \tag{2.9}$$

where the dual gauge coupling g is scaled as

$$g' \equiv \sqrt{\frac{3}{2}} g. \tag{2.10}$$

One finds that the dual gauge symmetry becomes very easy to observe, since the dual gauge transformation is defined by

$$\begin{aligned} &\chi_i \to \chi_i e^{if_i}, \quad \chi_i^* \to \chi_i^* e^{-if_i}, \\ &B^{\rm reg}_{i\ \mu} \to B^{\rm reg}_{i\ \mu} - \frac{1}{g'} \partial_\mu f_i, \qquad (i = 1, 2, 3) \end{aligned} \tag{2.11}$$

and accordingly the Lagrangian (2.8) has the extended local symmetry $[U(1)]^3_m$. However, it does not mean an increase of the gauge degrees of freedom because we have the constraint $\sum_{i=1}^{3} B_{i\ \mu} = 0$.

The field equations are given by

$$\left(\partial_\mu + ig' \left(B^{\rm reg}_{i\ \mu} + B^{\rm sing}_{i\ \mu} \right) \right)^2 \chi_i = -2\lambda \chi_i (\chi_i^* \chi_i - v^2), \tag{2.12}$$

$$\partial^\nu {}^*F^{\rm reg}_{i\ \mu\nu} \equiv k_{i\ \mu} = -ig' \left(\chi_i^* \partial_\mu \chi_i - \chi_i \partial_\mu \chi_i^* \right) + 2g'^2 \left(B^{\rm reg}_{i\ \mu} + B^{\rm sing}_{i\ \mu} \right) \chi_i^* \chi_i, \tag{2.13}$$

These field equations are to be solved with the proper boundary conditions that quantize the color-electric flux [8]. The flux is given by

$$\Phi_i \equiv \int {}^*F^{\rm reg}_{i\ \mu\nu} d\sigma^{\mu\nu} = \oint B^{\rm reg}_{i\ \mu} dx^\mu, \tag{2.14}$$

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[3] In order to classify the types of the flux-tube, we use the word such as the q-$\bar{q}$ system.

where $\sigma^{\mu\nu}$ is a two-dimensional surface element in the Minkowski space. By using the polar decomposition of the monopole field as $\chi_i = \phi_i e^{i\eta_i}$ ($\phi_i, \eta_i \in \Re$), we get, from the field equation (2.13),

$$B^{\rm reg}_{i\,\mu} = \frac{k_{i\,\mu}}{2g'^2\phi_i^2} - B^{\rm sing}_{i\,\mu} - \frac{1}{g'}\partial_\mu\eta_i. \tag{2.15}$$

We substitute this expression into (2.14) and integrate out over a large closed loop where the monopole current $k_{i\,\mu}$ is vanished. Thus we get

$$\Phi_i = -\oint\left(B^{\rm sing}_{i\,\mu} + \frac{1}{g'}\partial_\mu\eta_i\right)dx^\mu. \tag{2.16}$$

It is suggested from this expression that there are two possibilities to obtain the flux-tube configuration. One is originated from the singularity in $B^{\rm sing}_{i\,\mu}$ and the other is from the singularity in $\partial_\mu\eta_i$. We find that the former case, as can be seen from the relation (2.7), corresponds to the flux-tube which has the quark source. On the other hands, the latter case, it does not contain any information of the quark, which means no terminal, hence, it cannot provide the physical state like a q-$\bar{q}$ system. If one assumes the existence of the external color-electric source or the glueball state as the flux-tube ring [16], it should be taken into account. However, since this is not the case which we discuss in this paper, we assume that there is no singularity in $\partial_\mu\eta_i$. Then, this term can be absorbed into the regular dual gauge field $B^{\rm reg}_{i\,\mu}$ by the replacement $B^{\rm reg}_{i\,\mu} + \partial_\mu\eta_i/g' \rightarrow B^{\rm reg}_{i\,\mu}$. In this case, the flux (2.16) just has the meaning of the boundary condition of the regular dual gauge field which should behave as $B^{\rm reg}_{i\,\mu} \rightarrow -B^{\rm sing}_{i\,\mu}$ at infinity, where monopoles are condensed.

III. THE STATIC Q-$\bar{Q}$ SYSTEM

In this section, we consider the static q-$\bar{q}$ system. The quark source is given by the c-number current, which is typical in the heavy quark system,

$$\vec{j}^\mu \equiv \vec{j}^\mu_j(x) = \vec{Q}_j g^{\mu 0}\left[\delta^{(3)}(\boldsymbol{x}-\boldsymbol{a}) - \delta^{(3)}(\boldsymbol{x}-\boldsymbol{b})\right], \tag{3.1}$$

where $\vec{Q}_j \equiv e\vec{w}_j$ is the Abelian color-electric charge of the quark. Here, $\boldsymbol{a}$ and $\boldsymbol{b}$ are position vectors of the quark and the antiquark, respectively, and $\vec{w}_j$ is the weight vector of $SU(3)$ algebra, $\vec{w}_1 = \left(1/2, \sqrt{3}/6\right), \vec{w}_2 = \left(-1/2, \sqrt{3}/6\right), \vec{w}_3 = \left(0, -1/\sqrt{3}\right)$. This vector is nothing but the diagonal component of $\vec{H} = (T_3, T_8)$. The label $j = 1, 2, 3$ can be assigned to the charge red(R), blue(B) and green(G). We assume the cylindrical geometry of the system

by taking $\boldsymbol{a} = -(r/2)\boldsymbol{e}_z$, $\boldsymbol{b} = (r/2)\boldsymbol{e}_z$, and $n_\mu = \boldsymbol{e}_z$, where the distance between the quark and the anti-quark is defined by r. In this system, we get an explicit form of the singular dual gauge field from the relation (2.7) as

$$\boldsymbol{B}_i^{\rm sing} = \sqrt{\frac{2}{3}}\vec{\epsilon}_i \cdot \left[-\frac{\vec{Q}_j}{4\pi\rho}\left(\frac{z+r/2}{\sqrt{\rho^2+(z+r/2)^2}} - \frac{z-r/2}{\sqrt{\rho^2+(z-r/2)^2}}\right)\boldsymbol{e}_\varphi\right], \tag{3.2}$$

where φ is the azimuthal angle around the z-axis and ρ denotes the radial coordinate. Since the color-electric charges are defined on the weight vector of $SU(3)$ algebra, there arises the relation

$$\vec{\epsilon}_i \cdot \vec{w}_j = -\frac{1}{2}\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} = -\frac{1}{2}\sum_{k=1}^{3}\epsilon_{ijk} \equiv -\frac{1}{2}m_{ij}, \tag{3.3}$$

where m_{ij} takes 0 or $\pm$ 1. The zero of the diagonal component means that one of the monopole field is decoupled from the system and it does not contribute to the energy when we pay attention to the one of the color-electric charge, since the color-magnetic charge of the monopole field is defined on the root vector of $SU(3)$ algebra, as $g\vec{\epsilon}_i$.

Here, we investigate the ideal system for the limit $r \to \infty$. That is,

$$\lim_{r\to\infty}\boldsymbol{B}_i^{\rm sing} = \sqrt{\frac{2}{3}}\frac{em_{ij}}{4\pi\rho}\boldsymbol{e}_\varphi = \frac{m_{ij}}{g'\rho}\boldsymbol{e}_\varphi, \tag{3.4}$$

where we have used $eg = 4\pi$ and $g' = \sqrt{3/2}g$. Then, the fields depend only on the radial coordinate,

$$\phi_i = \phi_i(\rho), \quad \boldsymbol{B}_i^{\rm reg} = B_i^{\rm reg}(\rho)\boldsymbol{e}_\varphi \equiv \frac{\tilde{B}_i^{\rm reg}(\rho)}{\rho}\boldsymbol{e}_\varphi, \tag{3.5}$$

and the field equations (2.12) and (2.13) are reduced to

$$\frac{d^2\phi_i}{d\rho^2} + \frac{1}{\rho}\frac{d\phi_i}{d\rho} - \left(\frac{g'\tilde{B}_i^{\rm reg} + m_{ij}}{\rho}\right)^2\phi_i - 2\lambda\phi_i(\phi_i^2 - v^2) = 0, \tag{3.6}$$

$$\frac{d^2\tilde{B}_i^{\rm reg}}{d\rho^2} - \frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho} - 2g'\left(g'\tilde{B}_i^{\rm reg} + m_{ij}\right)\phi_i^2 = 0, \tag{3.7}$$

The string tension can be defined by the energy per unit length of the flux-tube,

$$\sigma = 2\pi\sum_{i=1}^{3}\int_0^\infty \rho d\rho\left[\frac{1}{2}\left(\frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho}\right)^2 + \left(\frac{d\phi_i}{d\rho}\right)^2 + \left(\frac{g'\tilde{B}_i^{\rm reg} + m_{ij}}{\rho}\right)^2\phi_i^2 + \lambda(\phi_i^2 - v^2)^2\right], \tag{3.8}$$

and we obtain the flux quantization condition,

$$\Phi_i = -\frac{2\pi m_{ij}}{g'}. \tag{3.9}$$

The boundary conditions are given by

$$\begin{aligned}
\tilde{B}_i^{\rm reg} &= 0, \quad \phi_i = \begin{cases} 0 \;\; (i \neq j) \\ v \;\; (i = j) \end{cases} \quad && \text{as} \quad \rho \to 0, \\
\tilde{B}_i^{\rm reg} &= -\frac{m_{ij}}{g'}, \quad \phi_i = v && \text{as} \quad \rho \to \infty.
\end{aligned} \tag{3.10}$$

Here, we shall confirm the relation (2.7). In this cylindrical system, the non-local term can be computed explicitly,

$$\begin{aligned}
\sqrt{\frac{2}{3}}\vec{\epsilon}_i \cdot \frac{1}{n\cdot\partial}\varepsilon_{\mu\nu\alpha\beta}n^\alpha \vec{j}^\beta &= \sqrt{\frac{2}{3}}\vec{\epsilon}_i \cdot -\vec{Q}_j\delta(x)\delta(y)\boldsymbol{e}_z \quad \left(\vec{Q}_j \equiv e\vec{w}_j\right) \\
&= \boldsymbol{\nabla} \times \left(\frac{m_{ij}}{g'\rho}\boldsymbol{e}_\varphi\right).
\end{aligned} \tag{3.11}$$

As can be seen from this expression, one finds that this term exactly cancels with the color-electric field which is originated from the singular dual gauge field $\boldsymbol{B}_i^{\rm sing}$ in (3.4). It shows that the kinetic term of the dual gauge field in the Lagrangian (2.8) can be written with no singular field.

IV. BOGOMOL'NYI LIMIT

In this section, we discuss the properties of the flux-tube in the Bogomol'nyi limit. Since now we have the same Lagrangian with $U(1)$ gauge symmetry except only the labels of i and j which classify the kinds of the monopole and the quark corresponding to $[U(1)]_m^3$ dual gauge symmetry, we can use the same strategy to find the Bogomol'nyi limit as given in Ref. [9]. Thus, we can write the string tension (3.8) exactly in the form,

$$\begin{aligned}
\sigma = 2\pi\sum_{i=1}^{3}|m_{ij}|v^2 + 2\pi\sum_{i=1}^{3}\int_0^\infty \rho d\rho\Bigg[\frac{1}{2}\left(\frac{1}{\rho}\frac{d\tilde{B}_i^{\rm reg}}{d\rho} \pm g'(\phi_i^2 - v^2)\right)^2 \\
+ \left(\frac{d\phi_i}{d\rho} \pm \left(g'\tilde{B}_i^{\rm reg} + m_{ij}\right)\frac{\phi_i}{\rho}\right)^2 + \frac{1}{2}\left(2\lambda - g'^2\right)(\phi_i^2 - v^2)^2\Bigg].
\end{aligned} \tag{4.1}$$

From this expression, we find the Bogomol'nyi limit,

$$g'^2 = 2\lambda, \quad \text{or} \quad 3g^2 = 4\lambda. \tag{4.2}$$

In this limit, one find that the string tension is reduced to

$$\sigma = 2\pi \sum_{i=1}^{3} |m_{ij}| v^2 = 4\pi v^2, \tag{4.3}$$

and the profiles of the dual gauge field and the monopole field is determined by the first order differential equations,

$$\frac{d\phi_i}{d\rho} \pm \left(g' \tilde{B}_i^{\rm reg} + m_{ij} \right) \frac{\phi_i}{\rho} = 0, \tag{4.4}$$

$$\frac{1}{\rho} \frac{d\tilde{B}_i^{\rm reg}}{d\rho} \pm g'(\phi_i^2 - v^2) = 0. \tag{4.5}$$

These field equations of cource reproduce the second order differential equations (3.6) and (3.7) when the relation (4.2) is satisfied.

Here, to obtain the string tension of the form (4.1) and the saturated string tension (4.3), we have paid attention to the boundary conditions of the fields (3.10) by taking into account the relation (3.3). For instance, let us consider the R-$\bar{R}$ flux-tube, which is given by the label $j = 1$. In this system, the monopole field ϕ_1 which has the magnetic charge $g\vec{\epsilon}_1$ is decoupled from the system, since ϕ_1 does not feel any singularity of the flux-tube core, and accordingly, the regular dual gauge field $B_1^{\rm reg}$ is also decoupled. The behavior of the other fields is interesting, ϕ_2 and ϕ_3 behaves as the same monopole field, and $B_2^{\rm reg}$ and $B_3^{\rm reg}$ provides the $U(1)_{i=2}$ flux-tube and $U(1)_{i=3}$ *anti* flux-tube due to the sign of the m_{ij}, which takes 1 and -1, respectively. Here, both dual gauge fields are related with each other through the constraint $\sum_{i=1}^{3} B_i^{\rm reg} = 0$, and $U(1)_{i=3}$ anti flux-tube can be regarded as the $U(1)_{i=2}$ flux-tube, or vise versa. As a result, these flux-tubes provide the same string tension $2\pi v^2$, and finally, we get two times of this string tension, $2 \times 2\pi v^2$. This is caused by the $[U(1)]_m^2$ dual gauge symmetry. We note that this discussion is the Weyl symmetric, and thus, the final expression for the string tension (4.3) does not depend on kind of the color-electric charges $\vec{Q}_j$. The profiles of the color-electric field can be obtained by solving the first order equations (4.4) and (4.5) by taking into account the above discussion as is discussed in Ref. [9,10].

Let us consider the meaning of (4.2). Here, we can define two characteristic scales using three parameters in the DGL theory, g, λ and v. One is the mass of the dual gauge field $m_B = \sqrt{2} g' v = \sqrt{3} g v$ and the other is the mass of the monopole field $m_\chi = 2\sqrt{\lambda} v$. These masses are extracted from the Lagrangian (2.8) by taking into account the dual Higgs mechanism. Thus, one finds that the Bogomol'nyi limit in the DGL theroy (4.2) is the

supersymmetry between the dual gauge field and the monopole field. Since these inverse masses m_B^{-1} and m_χ^{-1} corresponds to the penetration depth of the color-electric field and the coherent length of the monopole field, respectively, the Ginzburg-Landau (GL) parameter is defined:

$$\tilde{\kappa} \equiv \frac{m_B^{-1}}{m_\chi^{-1}} = \frac{\sqrt{2\lambda}}{g'} = \frac{2\sqrt{\lambda}}{\sqrt{3}g}. \tag{4.6}$$

Therefore, $\tilde{\kappa} = 1$ is regarded as the Bogomol'nyi limit, and the vacuum is classified into two types in terms of the Bogomol'nyi limit: $\tilde{\kappa} < 1$ belongs to the type-I vacuum and $\tilde{\kappa} > 1$ is the type-II vacuum.

Now, we would like to discuss the interaction between two parallel flux-tubes of the same type, such as the system R-$\bar{R}$ and R-$\bar{R}$. In general, the flux-tubes would interact with each other. However, in the Bogomol'nyi limit, there is no interaction between them. This can be understood through an investigtion of the generalized string tension for an exotica that the color-electric charges are given by $n\vec{Q}_j$ and $-n\vec{Q}_j$ for an integer n. In this system, we get the generalized string tension,

$$\sigma_n = 4\pi n v^2, \tag{4.7}$$

where m_{ij} is simply replaced to nm_{ij}. One finds that the string tension (4.7) is proportional to n, which implies that the interaction energy is zero. It is considered that this comes from the balance of propagation range of the dual gauge field and the monopole field since $m_B \sim m_\chi$. In the type-I or in the type-II vacuum, which is away from the Bogomol'nyi limit, the interaction range of these fields lose its balance, and the flux-tube interaction manifestly appears. The string tension is not proportional to n any more. While the attractive force is worked between two parallel flux-tubes in the type-I vacuum, the flux-tubes repel each other in the type-II vacuum. Numerical investigations of the interaction between two or more parallel flux-tubes of the same type in the DGL theory are given in Ref. [17,18].

It is interesting to investigate what happens if two parallel flux-tubes of different types are placed at a certain distance [19]. Here, according to the $[U(1)]_m^3$ dual gauge symmetry, there appear three different types of the flux-tube, such as given by R-$\bar{R}$, B-$\bar{B}$, and G-$\bar{G}$, so that these interactions seem to be very complicated. However, now the system has remarkable aspects owing to the Weyl symmetry. For instance, let us consider the interaction between R-$\bar{R}$ and B-$\bar{B}$. We find that the interaction between them is attractive, since if we suppose that these flux-tubes are unified into one flux-tube, it becomes $\bar{G}$-G (See the relation (3.3)).

It means that the energy of the system after unification is reduced into a half of the initial one. The same interaction property would be observed in the process, B-$\bar{B}$ + G-$\bar{G}$ $\rightarrow$ $\bar{R}$-R and G-$\bar{G}$ + R-$\bar{R}$ $\rightarrow$ $\bar{B}$-B. These investigations show that if we pay attention to the Weyl symmetry, we can easily obtain qualitative information about the flux-tube interaction.

V. CONCLUSION

We have studied the flux-tube solution in the DGL theory in the Bogomol'nyi limit by using the manifestly Weyl invariant form of the DGL Lagrangian. Here, the original dual gauge symmetry $[U(1)]_m^2$ is extended to $[U(1)]_m^3$. This replacement makes the further manipulation of the Lagrangian analogous to the $U(1)$ case. We have found that the Bogomol'nyi limit is given by $3g^2 = 4\lambda$, and the string tension is calculated as $\sigma_n = 4\pi n v^2$ for a q-$\bar{q}$ pair with the charge nQ_j and $-nQ_j$ in the both ends. In this limit, the mass of the dual gauge field and the monopole field becomes *exactly* the same. It should be noted that we could see the same relation with $U(1)$ Abelian Higgs theory except for three different types of the flux-tube. To summarize, the very similar properties with the ANO vortex in the Abelian Higgs theory is observed when we see the single flux-tube in the DGL theory, and the flux-tube solution can be easily obtained if we pay attention to the Weyl symmetry in the the DGL theory.

Finally, we would like to mention about the relation between the work in Ref. [11] and our study. If we replace the monopole field and the parameters that they have used as $\chi \rightarrow \sqrt{2}\chi$, $\eta \rightarrow \sqrt{2}v$, and $\lambda \rightarrow \lambda/4$, we get the same framework at the starting point, and the Bogomol'nyi limit is replaced as $3g^2 = 16\lambda \rightarrow 3g^2 = 4\lambda$. The idea of the extension of the dual gauge symmetry based on the Weyl symmetry in our case, however, seems to be simple to reach the final expression on the string tension, which can be applied to the $[U(1)]^{N-1}$ dual Abelian Higgs theroy reduced from the $SU(N)$ gluodynamics, straightforwardly.

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